FYJC - MATHEMATICS & STATISTICS

HIGHLIGHTS

- ✓ SOLUTION TO ALL QUESTIONS
- ✓ SOLUTIONS ARE PUT IN WAY THE STUDENT IS EXPECTED TO REPRODUCE IN THE BOARD EXAM
- ✓ TAUGHT IN THE CLASS ROOM THE SAME WAY AS THE SOLUTION ARE PUT UP HERE . THAT MAKES THE STUDENT TO EASILY GO THROUGH THE SOLUTION & PREPARE HIM/HERSELF WHEN HE/SHE SITS BACK TO REVISE AND RECALL THE TOPIC AT ANY GIVEN POINT OF TIME .
- ✓ LASTLY, IF STUDENT DUE TO SOME UNAVOIDABLE REASONS , HAS MISSED THE LECTURE , WILL NOT HAVE TO RUN HERE AND THERE TO UPDATE HIS/HER NOTES .
- ✓ HOWEVER STUDENT IS REQUESTED NOT TO MISUSE THE ABOVE POINT AS CLASS ROOM LECTURES ARE MUST FOR EASY PASSAGE OF UNDERSTANDING & LEARNING THE MINUEST DETAILS OF THE GIVEN TOPIC

PAPER - II

LOGARITHMS

Compiled & Conducted @ JKSC

Q. SET - 1

01. $\log 540 = 2 \log 2 + 3 \log 3 + \log 5$

02. $\log 360 = 3 \log 2 + 2 \log 3 + \log 5$

03. $\log \left(\frac{50}{147}\right) = \log 2 + 2 \log 5 - \log 3 - 2 \log 7$

04. $\log_{10} \left(\frac{12}{5}\right) + \log_{10} \left(\frac{25}{21}\right) - \log_{10}\left(\frac{2}{7}\right) = 1$

05. $\log_{10}\left(\frac{15}{16}\right) + \log_{10}\left(\frac{64}{81}\right) - \log_{10}\left(\frac{20}{27}\right) = 0$

06. $\log \left(\frac{75}{16}\right) - 2 \log \left(\frac{5}{9}\right) + \log \left(\frac{32}{243}\right) = \log 2$

07. $7 \log\left(\frac{16}{15}\right) + 5 \log \left(\frac{25}{24}\right) + 3 \log\left(\frac{81}{80}\right) = \log 2$

08. $7 \log \left(\frac{15}{16}\right) + 6 \log\left(\frac{8}{3}\right) + 5\log\left(\frac{2}{5}\right) + \log \left(\frac{32}{25}\right) = \log 3$

09. $4 \log_7 \left(\frac{3}{25}\right) + 3\log_7 \left(\frac{25}{7}\right) + 2\log_7\left(\frac{35}{9}\right) = -1$

10. $7 \log_2 \left(\frac{16}{15}\right) + 5\log_2 \left(\frac{25}{24}\right) + 3\log_2\left(\frac{81}{80}\right) = 1$

11. $\log_{10} 2 + 16 \log_{10}\left(\frac{16}{15}\right) + 12\log_{10}\left(\frac{25}{24}\right) + 7 \log_{10} \left(\frac{81}{80}\right) = 1$

12. $\log_{10} \left(\frac{351}{539}\right) + 2 \log_{10}\left(\frac{91}{110}\right) - 3\log_{10}\left(\frac{39}{110}\right) = 1$

Q. SET - 2

01. $\log\left(\frac{x + y}{7}\right) = \frac{1}{2} \left(\log x + \log y \right)$; Show that : $\frac{x}{y} + \frac{y}{x} = 47$

02. $\log\left(\frac{x + y}{3}\right) = \frac{1}{2} \left(\log x + \log y \right)$; Show that : $\frac{x}{y} + \frac{y}{x} = 7$

03. $\log\left(\frac{x-y}{4}\right) = \log\sqrt{x} + \log\sqrt{y}$; Show that : $(x+y)^2 = 20xy$

04. $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}\left(\log a + \log b\right)$; Show that : $a = b$

05. $\log(x+y) = \log 3 + \frac{1}{2}\log x + \frac{1}{2}\log y$; Show that : $x^2 + y^2 = 7xy$

06. if $a^2 + b^2 = 7ab$; Prove $2\log\left(\frac{a+b}{3}\right) = \log a + \log b$

07. if $x^2 + y^2 = 27xy$; Prove $\log\left(\frac{x-y}{5}\right) = \frac{1}{2}\left(\log x + \log y\right)$

08. if $a^2 + b^2 = 3ab$; Prove $\log\left(\frac{a+b}{\sqrt{5}}\right) = \frac{1}{2}\left(\log a + \log b\right)$

09. if $a^2 + b^2 = 14ab$; Prove $2\log(a+b) = 2\log 4 + \log a + \log b$

10. if $a^2 + b^2 = 11ab$; Prove $2\log(a-b) = 2\log 3 + \log a + \log b$

11. if $x^2 - xy + y^2 = 0$; Prove $\log(x+y) = \frac{1}{2}\left(\log x + \log y + \log 3\right)$

12. if $a^2 - 12ab + 4b^2 = 0$; Prove $\log(a+2b) = \frac{1}{2}\left(\log a + \log b\right) + 2\log 2$

13. if $a^3 + b^3 = ab(27 - 3a - 3b)$; Prove : $\log\left(\frac{a+b}{3}\right) = \frac{1}{3}\left(\log a + \log b\right)$

Q. SET - 3

01. $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$

then prove a. $x.y.z = 1$

b. $x^a . y^b . z^c = 1$

c. $x^{b+c} . y^{c+a} . z^{a+b} = 1$

d. $x^{b+c-a} . y^{c+a-b} . z^{a+b-c} = 1$

02. $\frac{\log x}{b + c - 2a} = \frac{\log y}{c + a - 2b} = \frac{\log z}{a + b - 2c}$ then prove $x.y.z = 1$

03. $\frac{\log x}{2} = \frac{\log y}{3} = \frac{\log z}{1}$ then prove $x.y^{-3}.z^{7} = 1$

04. $\frac{\log x}{1} = \frac{\log y}{3} = \frac{\log z}{7}$ then prove $x^{5}.y^{3}.z^{-2} = 1$

05. $\frac{\log x}{a} = \frac{\log y}{2} = \frac{\log z}{5}$ and $x^{4}.y^{3}.z^{-2} = 1$. Find a

06. $\frac{\log_2 a}{4} = \frac{\log_2 b}{6} = \frac{\log_2 c}{3k}$ and $a^{3}.b^{2}.c = 1$. Find k

Q. SET - 4

01. $\log_3 16 . \log_5 27 . \log_2 25 = 24$

02. $\log_5 16 . \log_7 125 . \log_4 49 = 12$

03. $\log_5 64 . \log_7 25 . \log_4 343 = 18$

04. $\log_b a^5 . \log_c b^3 . \log_a c^7 = 105$

05. $\log_y \sqrt{x} . \log_z y^3 . \log_x \sqrt[3]{z^2} = 1$

06. $\log_b \sqrt[3]{a} . \log_c b^4 . \log_a \sqrt[4]{c^3} = 1$

07. Prove : $(\log_4 2)(\log_2 3) = (\log_4 5)(\log_5 3)$

08. Prove that : $\frac{\log_2 7}{1 + \log_2 3} = \log_6 7$

09. Prove that : $\frac{\log_2 11}{2 + \log_2 3} = \log_{12} 11$

10. $\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = 1$

11. $\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc} = 2$

12. $\frac{1}{\log_a t} + \frac{1}{\log_b t} + \frac{1}{\log_c t} = \frac{1}{\log_z t}$ then prove z = abc

13. $\frac{1}{\log_2 30} + \frac{1}{\log_3 30} + \frac{1}{\log_5 30} = 1$

14. $\frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24} = 2$

15. $x = \log_a bc$; $y = \log_b ac$; $z = \log_c ab$ then Prove : $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+Z} = 1$

16. if $x = 1 + \log_a bc$; $y = 1 + \log_b ca$; $x = 1 + \log_c bc$

prove that : $xy + yz + zx = xyz$

17. $x = \log_6 3$; $y = \log_9 6$; $z = \log_{12} 9$ then prove that : $1 + xyx = 2yz$

18. $\log_a x + \log_{a^2} x^2 + \log_{a^3} x^3 +$ ______ $+ \log_{a^p} x^p = p \log_a x$

19. $\frac{1}{\log_{10} \frac{1}{30}} + \frac{1}{\log_5 \frac{1}{30}} + \frac{1}{\log_{18} \frac{1}{30}}$

20. if $\log_a b + \log_c b = 2 \log_a b . \log_c b$;then prove $b^2 = ac$

21. if $a^2 + c^2 = b^2$ then show that: $\frac{1}{\log_{b+c} a} + \frac{1}{\log_{b-c} a} = 2$

22. if $a^2 = b^3 = c^5 = d^6$ then show that $\log_d abc = \frac{31}{5}$

23. if $a^2 = b^3 = c^4 = d^5$ then show that $\log_a bcd = \frac{47}{30}$

24. if $a^x = b^y = c^z = d^w$ then show that $\log_a abcd = x\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)$

25. if $a^x = b^y = c^z$ and $b^2 = ac$ then prove that $y = \frac{2xz}{x+z}$

Q. SET - 5

SOLVE FOR 'X'

01. $\log(x + 3) + \log(x - 3) = \log 16$

02. $\log(3x + 2) - \log(3x - 2) = \log 5$

03. $\log 2 + \log(x + 3) - \log(3x - 5) = \log 3$

04. $\log_x(8x - 3) - \log_x 4 = 2$

05. $\log_x 3 + \log_x 8 + \log_x 6 = 2$

06. $\log_8 x + \log_4 x + \log_2 x = 11$

07. $\log_2 x + \log_4 x + \log_{16} x = 21/4$

08. $\log_3 x + \log_9 x + \log_{243} x = 34/5$

09. $\log_{\sqrt{3}} x + \log_3 x + \log_{\sqrt{27}} x = 11$

10. $2\log_{10} x = 1 + \log_{10}\left(x + \frac{11}{10}\right)$

11. $x + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$

12. $\log_2 x + \frac{1}{2}\log_2(x + 2) = 2$

13. $\sqrt{\log_2 x^4} + 4\log_4\sqrt{\frac{2}{x}} = 2$

Q. SET - 6

Without using log table prove :

01. $\frac{1}{4} < \log_{10} 2 < \frac{1}{3}$

02. $\frac{2}{5} < \log_{10} 3 < \frac{1}{2}$

03. $\frac{3}{10} < \log_{10} 2 < \frac{1}{3}$

04. $\frac{2}{3} < \log_{10} 5 < \frac{3}{4}$

SOLUTION TO - Q SET 1

01. $\log 540 = 2\log 2 + 3\log 3 + \log 5$

SOLUTION

RHS

$= 2\log 2 + 3\log 3 + \log 5$

$= \log 2^2 + \log 3^3 + \log 5$

$= \log 4 + \log 27 + \log 5$

$= \log (4 \times 27 \times 5)$

$= \log 540$ **= LHS**

02. $\log 360 = 3\log 2 + 2\log 3 + \log 5$

SOLUTION

RHS

$= 3\log 2 + 2\log 3 + \log 5$

$= \log 2^3 + \log 3^2 + \log 5$

$= \log 8 + \log 9 + \log 5$

$= \log (8 \times 9 \times 5)$

$= \log 360$ **= LHS**

03. $\log\left(\frac{50}{147}\right) = \log 2 + 2\log 5 - \log 3 - 2\log 7$

SOLUTION

$= \log 2 + 2\log 5 - \log 3 - 2\log 7$

$= \log 2 + \log 5^2 - \log 3 - \log 7^2$

$= \log 2 + \log 25 - \log 3 - \log 49$

$= \log\left(\frac{2 \times 25}{3 \times 49}\right)$

$= \log\left(\frac{50}{147}\right)$ **= LHS**

04. $\log_{10}\left(\frac{12}{5}\right) + \log_{10}\left(\frac{25}{21}\right) - \log_{10}\left(\frac{2}{7}\right) = 1$

SOLUTION

LHS

$= \log_{10}\left(\frac{12}{5}\right) + \log_{10}\left(\frac{25}{21}\right) - \log_{10}\left(\frac{2}{7}\right) = 1$

$= \log_{10}\left(\frac{\overset{6}{\cancel{12}} \times \overset{5}{\cancel{25}} \times \cancel{7}}{\cancel{5} \times \underset{3}{\cancel{21}} \times \cancel{2}}\right)$

$= \log_{10} 10$

$= 1$ **= RHS**

05. $\log_{10}\left(\frac{15}{16}\right) + \log_{10}\left(\frac{64}{81}\right) - \log_{10}\left(\frac{20}{27}\right) = 0$

SOLUTION

LHS

$= \log_{10}\left(\frac{15}{16}\right) + \log_{10}\left(\frac{64}{81}\right) - \log_{10}\left(\frac{20}{27}\right)$

$= \log_{10}\left(\frac{\cancel{15}^{3}}{\cancel{16}} \times \frac{\cancel{64}^{4}}{\cancel{81}_{3}} \times \frac{\cancel{27}}{\cancel{20}_{5}}\right)$

$= \log_{10} 1$

$= 0$ **= RHS**

06. $\log\left(\frac{75}{16}\right) - 2\log\left(\frac{5}{9}\right) + \log\left(\frac{32}{243}\right) = \log 2$

SOLUTION

LHS

$= \log\left(\frac{75}{16}\right) - 2\log\left(\frac{5}{9}\right) + \log\left(\frac{32}{243}\right)$

$= \log\left(\frac{75}{16}\right) - \log\left(\frac{5}{9}\right)^2 + \log\left(\frac{32}{243}\right)$

$= \log\left(\frac{75}{16}\right) - \log\left(\frac{25}{81}\right) + \log\left(\frac{32}{243}\right)$

$= \log\left(\frac{\cancel{75}^{3}}{\cancel{16}} \times \frac{\cancel{81}}{\cancel{25}} \times \frac{\cancel{32}^{2}}{\cancel{243}_{3}}\right)$

$= \log 2$ **= RHS**

07. $7\log\left(\frac{16}{15}\right) + 5\log\left(\frac{25}{24}\right) + 3\log\left(\frac{81}{80}\right) = \log 2$

SOLUTION

LHS

$= 7\log\left(\frac{2^4}{3.5}\right) + 5\log\left(\frac{5^2}{2^3.3}\right) + 3\log\left(\frac{3^4}{2^4.5}\right)$

$= \log\left(\frac{2^4}{3.5}\right)^7 + \log\left(\frac{5^2}{2^3.3}\right)^5 + \log\left(\frac{3^4}{2^4.5}\right)^3$

$= \log\left(\frac{2^{28}}{3^7.5^7}\right) + \log\left(\frac{5^{10}}{2^{15}.3^5}\right) + \log\left(\frac{3^{12}}{2^{12}.5^3}\right)$

$= \log\left(\frac{2^{28}}{3^7.5^7} \times \frac{5^{10}}{2^{15}.3^5} \times \frac{3^{12}}{2^{12}.5^3}\right)$

$= \log\left(\frac{2^{28} \times 3^{12} \times 5^{10}}{2^{27} \times 3^{12} \times 5^{10}}\right)$

$= \log 2$

08. $7\log\left(\frac{15}{16}\right) + 6\log\left(\frac{8}{3}\right) + 5\log\left(\frac{2}{5}\right) + \log\left(\frac{32}{25}\right) = \log 3$

SOLUTION

LHS

$= 7\log\left(\frac{15}{16}\right) + 6\log\left(\frac{8}{3}\right) + 5\log\left(\frac{2}{5}\right) + \log\left(\frac{32}{25}\right)$

$= 7\log\left(\frac{3.5}{2^4}\right) + 6\log\left(\frac{2^3}{3}\right) + 5\log\left(\frac{2}{5}\right) + \log\left(\frac{2^5}{5^2}\right)$ ------ PRIMES

$= \log\left(\frac{3.5}{2^4}\right)^7 + \log\left(\frac{2^3}{3}\right)^6 + \log\left(\frac{2}{5}\right)^5 + \log\left(\frac{2^5}{5^2}\right)$ ------ POWER UP

$= \log\left(\frac{3^7.5^7}{2^{28}}\right) + \log\left(\frac{2^{18}}{3^6}\right) + \log\left(\frac{2^5}{5^5}\right) + \log\left(\frac{2^5}{5^2}\right)$ ------ POWER IN

$= \log\left(\frac{3^7.5^7}{2^{28}} \times \frac{2^{18}}{3^6} \times \frac{2^5}{5^5} \times \frac{2^5}{5^2}\right)$ --------SARE LOG KA EK LOG

$= \log\left(\frac{2^{28} \times 3^7 \times 5^7}{2^{28} \times 3^6 \times 5^7}\right)$

$= \log 3$ **= RHS**

09. $4\log_7\left(\frac{3}{25}\right) + 3\log_7\left(\frac{25}{7}\right) + 2\log_7\left(\frac{35}{9}\right) = -1$

SOLUTION

LHS

$= 4\log_7\left(\frac{3}{25}\right) + 3\log_7\left(\frac{25}{7}\right) + 2\log_7\left(\frac{35}{9}\right)$

$= 4\log_7\left(\frac{3}{5^2}\right) + 3\log_7\left(\frac{5^2}{7}\right) + 2\log_7\left(\frac{5.7}{3^2}\right)$ ------ PRIMES

$= \log_7\left(\frac{3}{5^2}\right)^4 + \log_7\left(\frac{5^2}{7}\right)^3 + \log_7\left(\frac{5.7}{3^2}\right)^2$ ------ POWER UP

$= \log_7\left(\frac{3^4}{5^8}\right) + \log_7\left(\frac{5^6}{7^3}\right) + \log_7\left(\frac{5^2.7^2}{3^4}\right)$ ------ POWER IN

$= \log_7\left(\frac{3^4}{5^8} \times \frac{5^6}{7^3} \times \frac{5^2.7^2}{3^4}\right)$ --------SARE LOG KA EK LOG

$= \log_7\left(\frac{3^4 \times 5^8 \times 7^2}{3^4 \times 5^8 \times 7^3}\right)$

$= \log_7 7^{-1}$

$= -1 \log_7 7$

$= -1$

10. $7\log_2\left(\frac{16}{15}\right) + 5\log_2\left(\frac{25}{24}\right) + 3\log_2\left(\frac{81}{80}\right) = 1$

SOLUTION

LHS

$= 7\log_2\left(\frac{16}{15}\right) + 5\log_7\left(\frac{25}{24}\right) + 3\log_7\left(\frac{81}{80}\right)$

$= 7\log_2\left(\frac{2^4}{3.5}\right) + 5\log_2\left(\frac{5^2}{3.2^3}\right) + 3\log_2\left(\frac{3^4}{2^4.5}\right)$ ------ PRIMES

$= \log_2\left(\frac{2^4}{3.5}\right)^7 + \log_2\left(\frac{5^2}{3.2^3}\right)^5 + \log_2\left(\frac{3^4}{2^4.5}\right)^3$ ------ POWER UP

$= \log_2\left(\frac{2^{28}}{3^7.5^7}\right) + \log_2\left(\frac{5^{10}}{3^5.2^{15}}\right) + \log_2\left(\frac{3^{12}}{2^{12}.5^3}\right)$ ------ POWER IN

$= \log_2\left(\frac{2^{28}}{3^7.5^7} \times \frac{5^{10}}{3^5.2^{15}} \times \frac{3^{12}}{2^{12}.5^3}\right)$ --------SARE LOG KA EK LOG

$= \log_2\left(\frac{2^{28} \times 3^{12} \times 5^{10}}{2^{27} \times 3^{12} \times 5^{10}}\right) = \log_2 2 = 1 = \text{RHS}$

11. $\log_{10} 2 + 16 \log_{10}\left(\frac{16}{15}\right) + 12\log_{10}\left(\frac{25}{24}\right) + 7 \log_{10}\left(\frac{81}{80}\right) = 1$

SOLUTION

LHS

$$= \log_{10} 2 + 16 \log_{10}\left(\frac{16}{15}\right) + 12\log_{10}\left(\frac{25}{24}\right) + 7 \log_{10}\left(\frac{81}{80}\right)$$

$$= \log_{10} 2 + 16 \log_{10}\left(\frac{2^4}{3.5}\right) + 12\log_{10}\left(\frac{5^2}{3.2^3}\right) + 7 \log_{10}\left(\frac{3^4}{2^4.5}\right)$$

$$= \log_{10} 2 + \log_{10}\left(\frac{2^4}{3.5}\right)^{16} + \log_{10}\left(\frac{5^2}{3.2^3}\right)^{12} + \log_{10}\left(\frac{3^4}{2^4.5}\right)^7$$

$$= \log_{10} 2 + \log_{10}\left(\frac{2^{64}}{3^{16}5^{16}}\right) + \log_{10}\left(\frac{5^{24}}{3^{12}2^{36}}\right) + \log_{10}\left(\frac{3^{28}}{2^{28}5^7}\right)$$

$$= \log_{10}\left(2 \times \frac{2^{64}}{3^{16}5^{16}} \times \frac{5^{24}}{3^{12}2^{36}} \times \frac{3^{28}}{2^{28}5^7}\right)$$

$$= \log_{10}\left(\frac{2^{65} \times 3^{28} \times 5^{24}}{2^{64} \times 3^{28} \times 5^{23}}\right)$$

$$= \log_{10} 2.5$$

$$= \log_{10} 10$$

$= 1$ **= RHS**

12. $\log_{10}\left(\frac{351}{539}\right) + 2 \log_{10}\left(\frac{91}{110}\right) - 3\log_{10}\left(\frac{39}{110}\right) = 1$

SOLUTION

LHS

$$= \log_{10}\left(\frac{351}{539}\right) + 2 \log_{10}\left(\frac{91}{110}\right) - 3\log_{10}\left(\frac{39}{110}\right)$$

3	351
3	117
3	39
13	13
	1

7	539
7	77
11	11
	1

7	91
13	13
	1

$$= \log_{10}\left(\frac{3^3.13}{7^2.11}\right) + 2\log_{10}\left(\frac{7.13}{11.2.5}\right) - 3\log_{10}\left(\frac{3.13}{11.2.5}\right)$$

$$= \log_{10}\left(\frac{3^3.13}{7^2.11}\right) + \log_{10}\left(\frac{7.13}{11.2.5}\right)^2 - \log_{10}\left(\frac{3.13}{11.2.5}\right)^3$$

$$= \log_{10}\left(\frac{3^3.13}{7^2.11}\right) + \log_{10}\left(\frac{7^2.13^2}{11^2.2^2.5^2}\right) - \log_{10}\left(\frac{3^3.13^3}{11^3.2^3.5^3}\right)$$

$$= \log_{10}\left(\frac{3^3.13}{7^2.11} \times \frac{7^2.13^2}{11^2.2^2.5^2} \times \frac{11^3.2^3.5^3}{3^3.13^3}\right)$$

$$= \log_{10}\left(\frac{2^3 \times 3^3 \times 5^3 \times 11^3 \times 13^3}{2^2 \times 3^3 \times 5^2 \times 11^3 \times 13^3}\right)$$

$$= \log_{10} 2.5$$

$$= \log_{10} 10$$

$$= 1$$

SOLUTION TO - Q SET 2

01. $\log\left(\frac{x+y}{7}\right) = \frac{1}{2}\ \log x + \log y$

Show that : $\frac{x}{y} + \frac{y}{x} = 47$

SOLUTION :

$$\log\left(\frac{x+y}{7}\right) = \frac{1}{2}\left(\log x + \log y\right)$$

$$2\log\left(\frac{x+y}{7}\right) = \log x + \log y$$

$$\log\left(\frac{x+y}{7}\right)^2 = \log xy$$

$$\left(\frac{x+y}{7}\right)^2 = xy$$

$$\frac{x^2 + 2xy + y^2}{49} = xy$$

$$x^2 + 2xy + y^2 = 49xy$$

$$x^2 + y^2 = 47xy$$

Dividing throughout by xy

$$\frac{x^2}{xy} + \frac{y^2}{xy} = \frac{47xy}{xy}$$

$$\frac{x}{y} + \frac{y}{x} = 47$$

……. PROVED

02. $\log\left(\frac{x+y}{3}\right) = \frac{1}{2}\ \log x + \log y$

Show that : $\frac{x}{y} + \frac{y}{x} = 7$

SOLUTION :

$$\log\left(\frac{x+y}{3}\right) = \frac{1}{2}\left(\log x + \log y\right)$$

$$2\log\left(\frac{x+y}{3}\right) = \log x + \log y$$

$$\log\left(\frac{x+y}{3}\right)^2 = \log xy$$

$$\left(\frac{x+y}{3}\right)^2 = xy$$

$$\frac{x^2 + 2xy + y^2}{9} = xy$$

$$x^2 + 2xy + y^2 = 9xy$$

$$x^2 + y^2 = 7xy$$

Dividing throughout by xy

$$\frac{x^2}{xy} + \frac{y^2}{xy} = \frac{7xy}{xy}$$

$$\frac{x}{y} + \frac{y}{x} = 7$$

……. PROVED

03. $\log\left(\frac{x-y}{4}\right) = \log\sqrt{x} + \log\sqrt{y}$

Show that : $(x+y)^2 = 20xy$

SOLUTION :

$$\log\left(\frac{x-y}{4}\right) = \log\sqrt{x} + \log\sqrt{y}$$

$$\log\left(\frac{x-y}{4}\right) = \log\sqrt{x}.\sqrt{y}$$

$$\left(\frac{x-y}{4}\right) = \sqrt{x}.\sqrt{y}$$

Squaring both sides

$$\left(\frac{x-y}{4}\right)^2 = xy$$

$$\frac{x^2 - 2xy + y^2}{16} = xy$$

$$x^2 - 2xy + y^2 = 16xy$$

$$x^2 + y^2 = 18xy$$

Adding '2xy' on both sides

$$x^2 + 2xy + y^2 = 20xy$$

$$(x+y)^2 = 20xy$$ ……. PROVED

04. $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}\left(\log a + \log b\right)$

Show that : a = b

SOLUTION :

$\log\left(\frac{a+b}{2}\right) = \frac{1}{2}\left(\log a + \log b\right)$

$2\log\left(\frac{a+b}{2}\right) = \log a + \log b$

$\log\left(\frac{a+b}{2}\right)^2 = \log ab$

$\left(\frac{a+b}{2}\right)^2 = ab$

$\frac{a^2 + 2ab + b^2}{4} = ab$

$a^2 + 2ab + b^2 = 4ab$

$a^2 + b^2 = 2ab$

$a^2 - 2ab + b^2 = 0$

$(a - b)^2 = 0$

$a - b = 0$

$a = b$ ……. PROVED

05. $\log(x + y) = \log 3 + \frac{1}{2}\log x + \frac{1}{2}\log y$

Show that : $x^2 + y^2 = 7xy$

SOLUTION :

$\log(x + y) = \frac{2\log 3 + \log x + \log y}{2}$

$2\log(x + y) = 2\log 3 + \log x + \log y$

$\log(x + y)^2 = \log 3^2 + \log x + \log y$

$\log(x + y)^2 = \log 9 + \log x + \log y$

$\log(x + y)^2 = \log 9xy$

$(x + y)^2 = 9xy$

$x^2 + 2xy + y^2 = 9xy$

$x^2 - y^2 = 7xy$ ……. PROVED

06. if $a^2 + b^2 = 7ab$

Prove $2\log\left(\frac{a+b}{3}\right) = \log a + \log b$

SOLUTION :

$a^2 + b^2 = 7ab$

Adding '2ab' on both sides

$a^2 + 2ab + b^2 = 9ab$

$(a + b)^2 = 9ab$

$\left(\frac{a+b}{3}\right)^2 = ab$

Inserting log on both sides

$\log\left(\frac{a+b}{3}\right)^2 = \log(ab)$

$2\log\left(\frac{a+b}{3}\right) = \log a + \log b$

……. PROVED

07. if $x^2 + y^2 = 27xy$

Prove $\log\left(\frac{x-y}{5}\right) = \frac{1}{2}\left(\log x + \log y\right)$

SOLUTION :

$x^2 + y^2 = 27xy$

Adding '–2xy' on both sides

$x^2 - 2xy + y^2 = 27xy - 2xy$

$(x - y)^2 = 25xy$

$\left(\frac{x-y}{5}\right)^2 = xy$

Inserting log on both sides

$\log\left(\frac{x-y}{5}\right)^2 = \log(xy)$

$2\log\left(\frac{x-y}{5}\right) = \log x + \log y$

$\log\left(\frac{x-y}{5}\right) = \frac{1}{2}\left(\log x + \log y\right)$

……. PROVED

08. if $a^2 + b^2 = 3ab$

Prove $\log\left(\frac{a+b}{\sqrt{5}}\right) = \frac{1}{2}\left(\log a + \log b\right)$

SOLUTION :

$a^2 + b^2 = 3ab$

Adding '2ab' on both sides

$a^2 + 2ab + b^2 = 3ab + 2ab$

$(a + b)^2 = 5ab$

$\left(\frac{a+b}{\sqrt{5}}\right)^2 = ab$

Inserting log on both sides

$\log\left(\frac{a+b}{\sqrt{5}}\right)^2 = \log ab$

$2\log\left(\frac{a+b}{\sqrt{5}}\right) = \log a + \log b$

$\log\left(\frac{a+b}{\sqrt{5}}\right) = \frac{1}{2}\left(\log a + \log b\right)$

……. PROVED

09. if $a^2 + b^2 = 14ab$ prove ;

$2\log(a + b) = 2\log 4 + \log a + \log b$

SOLUTION :

$a^2 + b^2 = 14ab$

Adding '2ab' on both sides

$a^2 + 2ab + b^2 = 14ab + 2ab$

$(a + b)^2 = 16ab$

Inserting log on both sides

$\log(a + b)^2 = \log 16ab$

$2\log(a + b) = \log 16 + \log a + \log b$

$2\log(a + b) = \log 4^2 + \log a + \log b$

$2\log(a + b) = 2\log 4 + \log a + \log b$

……. PROVED

10. if $a^2 + b^2 = 11ab$ prove ;

$2\log(a - b) = 2\log 3 + \log a + \log b$

SOLUTION :

$a^2 + b^2 = 14ab$

Adding '–2ab' on both sides

$a^2 - 2ab + b^2 = 11ab - 2ab$

$(a - b)^2 = 9ab$

Inserting log on both sides

$\log(a - b)^2 = \log 9ab$

$2\log(a - b) = \log 9 + \log a + \log b$

$2\log(a - b) = \log 3^2 + \log a + \log b$

$2\log(a - b) = 2\log 3 + \log a + \log b$

……. PROVED

11. if $x^2 - xy + y^2 = 0$ prove ;

$\log(x+y) = \frac{1}{2}\left(\log x + \log y + \log 3\right)$

SOLUTION :

$x^2 - xy + y^2 = 0$

$x^2 + y^2 = xy$

Adding '2xy' on both sides

$x^2 + 2xy + y^2 = xy + 2xy$

$(x + y)^2 = 3xy$

Inserting log on both sides

$\log(x + y)^2 = \log 3xy$

$2\log(x + y) = \log 3 + \log x + \log y$

$\log(x + y) = \frac{1}{2}\left(\log 3 + \log x + \log y\right)$

……. PROVED

12. if $a^2 - 12ab + 4b^2 = 0$

prove that

$\log(a + 2b) = \frac{1}{2}\left(\log a + \log b\right) + 2\log 2$

SOLUTION :

$a^2 - 12ab + 4b^2 = 0$

$a^2 + 4b^2 = 12ab$

ADDING '+4ab' ON BOTH SIDES

$a^2 + 4ab + b^2 = 12ab + 4ab$

$(a + 2b)^2 = 16ab$

INSERTING LOG ON BOTH SIDES

$\log (a + 2b)^2 = \log 16ab$

$2\log (a + 2b) = \log 16 + \log a + \log b$

$2\log (a + 2b) = \log 2^4 + \log a + \log b$

$2\log (a + 2b) = 4\log 2 + \log a + \log b$

$\log (a + 2b) = \frac{4\log 2 + \log a + \log b}{2}$

$\log (a + 2b) = \frac{4\log 2}{2} + \frac{1}{2}\left(\log a + \log b\right)$

$\log (a + 2b) = 2\log 2 + \frac{1}{2}\left(\log a + \log b\right)$

……. PROVED

13. if $a^3 + b^3 = ab(27 - 3a - 3b)$

prove that

$\log\left(\frac{a + b}{3}\right) = \frac{1}{3}\left(\log a + \log b\right)$

SOLUTION :

$a^3 + b^3 = ab(27 - 3a - 3b)$

$a^3 + b^3 = 27ab - 3a^2b - 3ab^2$

$a^3 + 3a^2b + 3ab^2 + b^3 = 27ab$

$(a + b)^3 = 27ab$

$\left(\frac{a + b}{3}\right)^3 = ab$

INSERTING LOG ON BOTH SIDES

$\log\left(\frac{a + b}{3}\right)^3 = \log ab$

$3\log\left(\frac{a + b}{3}\right) = \log a + \log b$

$\log\left(\frac{a + b}{3}\right) = \frac{1}{3}\left(\log a + \log b\right)$

……. PROVED

SOLUTION TO - Q SET 3

01.a) $\dfrac{\log x}{b-c} = \dfrac{\log y}{c-a} = \dfrac{\log z}{a-b}$ **PROVE : $x \cdot y \cdot z = 1$**

SOLUTION :

$\dfrac{\log x}{b-c} = \dfrac{\log y}{c-a} = \dfrac{\log z}{a-b} = k$

$\therefore \log_e x = k(b-c) \quad \therefore x = e^{k(b-c)}$

$\log_e y = k(c-a) \quad \therefore y = e^{k(c-a)}$

$\log_e z = k(a-b) \quad \therefore z = e^{k(a-b)}$

LHS

$= x \cdot y \cdot z$

$= e^{k(b-c)} \cdot e^{k(c-a)} \cdot e^{k(a-b)}$

$= e^{k(b-c) + k(c-a) + k(a-b)}$

$= e^{k(b-c+c-a+a-b)}$

$= e^{k(0)}$

$= e^0 = 1$ RHS

01.b) $\dfrac{\log x}{b-c} = \dfrac{\log y}{c-a} = \dfrac{\log z}{a-b}$ **PROVE : $x^a \cdot y^b \cdot z^c = 1$**

SOLUTION :

$\dfrac{\log x}{b-c} = \dfrac{\log y}{c-a} = \dfrac{\log z}{a-b} = k$

$\therefore \log_e x = k(b-c) \quad \therefore x = e^{k(b-c)}$

$\log_e y = k(c-a) \quad \therefore y = e^{k(c-a)}$

$\log_e z = k(a-b) \quad \therefore z = e^{k(a-b)}$

LHS

$= x^a \cdot y^b \cdot z^c$

$= e^{k(b-c)a} \cdot e^{k(c-a)b} \cdot e^{k(a-b)c}$

$= e^{k(ab-ac)} \cdot e^{k(bc-ab)} \cdot e^{k(ac-bc)}$

$= e^{k(ab-ac) + k(bc-ab) + k(ac-bc)}$

$= e^{k(ab-ac+bc-ab+ac-bc)}$

$= e^{k(0)} = e^0 = 1$ RHS

01.c) $\dfrac{\log x}{b-c} = \dfrac{\log y}{c-a} = \dfrac{\log z}{a-b}$ **PROVE : $x^{b+c} \cdot y^{c+a} \cdot z^{a+b} = 1$**

SOLUTION :

$\dfrac{\log x}{b-c} = \dfrac{\log y}{c-a} = \dfrac{\log z}{a-b} = k$

$\therefore \log_e x = k(b-c) \quad \therefore x = e^{k(b-c)}$

$\log_e y = k(c-a) \quad \therefore y = e^{k(c-a)}$

$\log_e z = k(a-b) \quad \therefore z = e^{k(a-b)}$

LHS

$= x^{b+c} \cdot y^{c+a} \cdot z^{a+b}$

$= e^{k(b-c)(b+c)} \cdot e^{k(c-a)(c+a)} \cdot e^{k(a-b)(a+b)}$

$= e^{k(b^2-c^2)} \cdot e^{k(c^2-a^2)} \cdot e^{k(a^2-b^2)}$

$= e^{k(b^2-c^2) + k(c^2-a^2) + k(a^2-b^2)}$

$= e^{k(b^2-c^2+c^2-a^2+a^2-b^2)}$

$= e^{k(0)}$

$= e^0 = 1$ RHS

02. $\dfrac{\log x}{b+c-2a} = \dfrac{\log y}{c+a-2b} = \dfrac{\log z}{a+b-2c}$ **PROVE :** $x.y.z = 1$

SOLUTION :

$\dfrac{\log x}{b+c-2a} = \dfrac{\log y}{c+a-2b} = \dfrac{\log z}{a+b-2c} = k$

$\therefore \log_e x = k(b+c-2a) \quad \therefore x = e^{k(b+c-2a)}$

$\log_e y = k(c+a-2b) \quad \therefore y = e^{k(c+a-2b)}$

$\log_e z = k(a+b-2c) \quad \therefore z = e^{k(a+b-2c)}$

LHS

$= x\,.\,y\,.z$

$= e^{k(b+c-2a)}\,.\,e^{k(c+a-2b)}\,.\,e^{k(a+b-2c)}$

$= e^{k(b+c-2a)+k(c+a-2b)+k(a+b-2c)}$

$= e^{k(b+c-2a+c+a-2b+a+b-2c)}$

$= e^{k(2a+2b+2c-2a-2b-2c)}$

$= e^{k(0)} \quad = e^0 \quad = 1$ RHS

03. $\dfrac{\log x}{2} = \dfrac{\log y}{3} = \dfrac{\log z}{1}$ **PROVE :** $x.y^{-3}.z^{7} = 1$

SOLUTION

$\dfrac{\log x}{2} = \dfrac{\log y}{3} = \dfrac{\log z}{1} = k$

$\log_e x = 2k \quad \therefore x = e^{2k}.$

$\log_e y = 3k \quad \therefore y = e^{3k}.$

$\log_e z = k \quad \therefore z = e^{k}.$

LHS $= x.y^{-3}.z^{7}$

$= e^{2k}.\,e^{3k.\,-3}.\,e^{k.7}$

$= e^{2k}.\,e^{-9k}.\,e^{7k}$

$= e^{2k-9k+7k}$

$= e^{0}$

$= 1 \quad = $ RHS

04. $\dfrac{\log x}{1} = \dfrac{\log y}{3} = \dfrac{\log z}{7}$ **PROVE :** $x^{5}.y^{3}.z^{-2} = 1$

SOLUTION

$\dfrac{\log x}{1} = \dfrac{\log y}{3} = \dfrac{\log z}{7} = k$

$\log_e x = k \quad \therefore x = e^{k}.$

$\log_e y = 3k \quad \therefore y = e^{3k}.$

$\log_e z = 7k \quad \therefore z = e^{7k}.$

LHS $= x^{5}.y^{3}.z^{7}$

$= e^{k.5}.\,e^{3k\,.3}.\,e^{7k.-2}$

$= e^{5k}.\,e^{9k}.\,e^{-14k}$

$= e^{5k+9k-14k}$

$= e^{0}$

$= 1 \quad = $ RHS

05. $\frac{\log x}{a} = \frac{\log y}{2} = \frac{\log z}{5}$ **GIVEN :** $x^4.y^3.z^{-2} = 1$; **find 'a'**

SOLUTION

$\frac{\log x}{a} = \frac{\log y}{2} = \frac{\log z}{5} = k$

$\log_e x = ak \quad \therefore \ x = e^{ak}.$

$\log_e y = 2k \quad \therefore \ y = e^{2k}.$

$\log_e z = 5k \quad \therefore \ z = e^{5k}.$

$x^4.y^3.z^{-2} = 1$

$e^{ak.4}.\ e^{2k\ .3}.\ e^{5k.-2} = 1$

$e^{4ak}.\ e^{6k}.\ e^{-10k} = 1$

$e^{4ak\ +\ 6k\ -\ 10k} = 1$

$e^{4ak\ -\ 4k} = e^0$

EQUATING THE POWERS

$4ak - 4k = 0$

$4ak = 4k \qquad a = 1$

06. $\frac{\log_2 a}{4} = \frac{\log_2 b}{6} = \frac{\log_2 c}{3k}$ **GIVEN :** $a^3.b^2.c = 1$. **Find k**

SOLUTION

$\frac{\log_2 a}{4} = \frac{\log_2 b}{6} = \frac{\log_2 c}{3k} = m$

$\log_2 a = 4m \quad \therefore \ a = 2^{4m}.$

$\log_2 b = 6m \quad \therefore \ b = 2^{6m}.$

$\log_2 c = 3km \quad \therefore \ c = 2^{3km}.$

$a^3.b^2.c = 1$

$2^{4m.3}.\ 2^{6m\ .2}.\ 2^{3km} = 1$

$2^{12m}.\ 2^{12m}.\ 2^{3km} = 1$

$2^{12m\ +\ 12m\ +\ 3km} = 1$

$2^{24m\ +\ 3km} = 2^0$

EQUATING THE POWERS

$24m + 3km = 0$

$3km = -24m$

$3k = -24$

$k = -8$

SOLUTION TO - Q SET 4

01. $\log_3 16 \cdot \log_5 27 \cdot \log_2 25 = 24$

SOLUTION

LHS

$= \log_3 16 \cdot \log_5 27 \cdot \log_2 25$

$= \frac{\log 16}{\log 3} \cdot \frac{\log 27}{\log 5} \cdot \frac{\log 25}{\log 2}$

$= \frac{\log 2^4}{\log 3} \cdot \frac{\log 3^3}{\log 5} \cdot \frac{\log 5^2}{\log 2}$

$= 4 \frac{\log 2}{\log 3} \cdot 3 \frac{\log 3}{\log 5} \cdot 2 \frac{\log 5}{\log 2}$

$= 4 \cdot 3 \cdot 2$

$= 24 \quad = \quad$ RHS

02. $\log_5 16 \cdot \log_7 125 \cdot \log_4 49 = 12$

SOLUTION

LHS

$= \log_5 16 \cdot \log_7 125 \cdot \log_4 49$

$= \frac{\log 16}{\log 5} \cdot \frac{\log 125}{\log 7} \cdot \frac{\log 49}{\log 4}$

$= \frac{\log 4^2}{\log 5} \cdot \frac{\log 5^3}{\log 7} \cdot \frac{\log 7^2}{\log 4}$

$= 2 \frac{\log 4}{\log 5} \cdot 3 \frac{\log 5}{\log 7} \cdot 2 \frac{\log 7}{\log 4}$

$= 2 \cdot 3 \cdot 2$

$= 12 \quad = \quad$ RHS

03. $\log_5 64 \cdot \log_7 25 \cdot \log_4 343 = 18$

SOLUTION

LHS

$= \log_5 64 \cdot \log_7 25 \cdot \log_4 343$

$= \frac{\log 64}{\log 5} \cdot \frac{\log 25}{\log 7} \cdot \frac{\log 343}{\log 4}$

$= \frac{\log 4^3}{\log 5} \cdot \frac{\log 5^2}{\log 7} \cdot \frac{\log 7^3}{\log 4}$

$= 3 \frac{\log 4}{\log 5} \cdot 2 \frac{\log 5}{\log 7} \cdot 3 \frac{\log 7}{\log 4}$

$= 3 \cdot 2 \cdot 3$

$= 18 \quad = \quad$ RHS

04. $\log_b a^5 \cdot \log_c b^3 \cdot \log_a c^7 = 105$

SOLUTION

LHS

$= \log_b a^5 \cdot \log_c b^3 \cdot \log_a c^7$

$= 5 \cdot \log_b a \cdot 3 \log_c b \cdot 7 \log_a c$

$= 5 \frac{\log a}{\log b} \cdot 3 \frac{\log b}{\log c} \cdot 7 \frac{\log c}{\log a}$

$= 5 \cdot 3 \cdot 7$

$= 105 \quad = \quad$ RHS

05. $\log_y \sqrt{x} \cdot \log_z y^3 \cdot \log_x \sqrt[3]{z^2} = 1$

SOLUTION

LHS

$= \log_y \sqrt{x} \cdot \log_z y^3 \cdot \log_x \sqrt[3]{z^2}$

$= \log_y x^{1/2} \cdot \log_z y^3 \cdot \log_x z^{2/3}$

$= \frac{1}{2} \log_y x \cdot 3 \log_z y \cdot \frac{2}{3} \log_x z$

$= \frac{1}{2} \frac{\log x}{\log y} \cdot 3 \frac{\log y}{\log z} \cdot \frac{2}{3} \frac{\log z}{\log x}$

$= \frac{1}{2} \cdot 3 \cdot \frac{2}{3}$

$= 1 \quad = \quad$ RHS

06. $\log_b \sqrt[3]{a} \cdot \log_c b^4 \cdot \log_a \sqrt[4]{c^3} = 1$

SOLUTION

LHS

$= \log_b \sqrt[3]{a} \cdot \log_c b^4 \cdot \log_a \sqrt[4]{c^3}$

$= \log_b a^{1/3} \cdot \log_c b^4 \cdot \log_a c^{3/4}$

$= \frac{1}{3}\log_b a \cdot 4 \log_c b \cdot \frac{3}{4} \log_a c$

$= \frac{1}{3} \frac{\log a}{\log b} \cdot 4 \frac{\log b}{\log c} \cdot \frac{3}{4} \frac{\log c}{\log a}$

$= \frac{1}{3} \cdot 4 \cdot \frac{3}{4}$

$= 1 \quad = \quad$ RHS

07. Prove :

$(\log_4 2)(\log_2 3) = (\log_4 5)(\log_5 3)$

SOLUTION

LHS $= (\log_4 2)(\log_2 3)$

$= \frac{\log 2}{\log 4} \cdot \frac{\log 3}{\log 2}$

$= \frac{\log 2}{\log 2^2} \cdot \frac{\log 3}{\log 2}$

$= \frac{\log 2}{2 \log 2} \cdot \frac{\log 3}{\log 2}$

$= \frac{\log 3}{2\log 2}$

RHS $= (\log_4 5)(\log_5 3)$

$= \frac{\log 5}{\log 4} \cdot \frac{\log 3}{\log 5}$

$= \frac{\log 3}{\log 2^2}$

$= \frac{\log 3}{2 \log 2}$

$= \frac{\log 3}{2\log 2}$ LHS = RHS

08. $\frac{\log_2 7}{1 + \log_2 3} = \log_6 7$

SOLUTION

LHS

$= \frac{\frac{\log 7}{\log 2}}{1 + \frac{\log 3}{\log 2}}$

$= \frac{\frac{\log 7}{\log 2}}{\frac{\log 2 + \log 3}{\log 2}}$

$= \frac{\log 7}{\log 2 + \log 3}$

$= \frac{\log 7}{\log 6}$

$= \log_6 7$

= RHS

09. $\frac{\log_2 11}{2 + \log_2 3} = \log_{12} 11$

SOLUTION

LHS

$= \frac{\frac{\log 11}{\log 2}}{2 + \frac{\log 3}{\log 2}}$

$= \frac{\frac{\log 11}{\log 2}}{\frac{2 \log 2 + \log 3}{\log 2}}$

$= \frac{\log 11}{2 \log 2 + \log 3}$

$= \frac{\log 11}{\log 2^2 + \log 3}$

$= \frac{\log 11}{\log 4 + \log 3}$

$= \dfrac{\log 11}{\log 12}$

$= \log_{12} 11$

= RHS

10. $\dfrac{1}{\log_a abc} + \dfrac{1}{\log_b abc} + \dfrac{1}{\log_c abc} = 1$

SOLUTION

LHS

$= \dfrac{1}{\log_a abc} + \dfrac{1}{\log_b abc} + \dfrac{1}{\log_c abc}$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} abc$

= 1

= RHS

11. $\dfrac{1}{\log_{ab} abc} + \dfrac{1}{\log_{bc} abc} + \dfrac{1}{\log_{ca} abc} = 2$

SOLUTION

LHS

$= \dfrac{1}{\log_{ab} abc} + \dfrac{1}{\log_{bc} abc} + \dfrac{1}{\log_{ca} abc}$

$= \log_{abc} ab + \log_{abc} bc + \log_{abc} ca$

$= \log_{abc} ab.bc.ca$

$= \log_{abc} a^2.b^2.c^2$

$= \log_{abc} (a.b.c)^2$

$= 2\log_{abc} abc$

= 2(1)

= 2

= RHS

12. $\dfrac{1}{\log_a t} + \dfrac{1}{\log_b t} + \dfrac{1}{\log_c t} = \dfrac{1}{\log_z t}$

then prove z = abc

SOLUTION

$\dfrac{1}{\log_a t} + \dfrac{1}{\log_b t} + \dfrac{1}{\log_c t} = \dfrac{1}{\log_z t}$

$\log_t a + \log_t b + \log_t c = \log_t z$

$\log_t abc = \log_t z$

$\therefore$ abc = z ……. PROVED

13. $\dfrac{1}{\log_2 30} + \dfrac{1}{\log_3 30} + \dfrac{1}{\log_5 30} = 1$

SOLUTION

LHS

$= \dfrac{1}{\log_2 30} + \dfrac{1}{\log_3 30} + \dfrac{1}{\log_5 30}$

$= \log_{30} 2 + \log_{30} 3 + \log_{30} 5$

$= \log_{30} 2.3.5$

$= \log_{30} 30$

= 1

= RHS

14. $\dfrac{1}{\log_6 24} + \dfrac{1}{\log_{12} 24} + \dfrac{1}{\log_8 24} = 2$

SOLUTION

LHS

$= \dfrac{1}{\log_6 24} + \dfrac{1}{\log_{12} 24} + \dfrac{1}{\log_8 24}$

$= \log_{24} 6 + \log_6 12 + \log_{24} 8$

$= \log_{24} 6.12.8$

$= \log_{24} 576$

$= \log_{24} 24^2$

$= 2 \log_{24} 24$

$= 2(1) = 2 = \text{RHS}$

15. $x = \log_a bc$; $y = \log_b ac$; $z = \log_c ab$

Prove : $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} = 1$

SOLUTION

LHS

$= \frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$

$= \frac{1}{1+\log_a bc} + \frac{1}{1+\log_b ac} + \frac{1}{1+\log_c ab}$

$= \frac{1}{1+\frac{\log bc}{\log a}} + \frac{1}{1+\frac{\log ac}{\log b}} + \frac{1}{1+\frac{\log ab}{\log c}}$

$= \frac{1}{\frac{\log a + \log bc}{\log a}} + \frac{1}{\frac{\log b + \log ac}{\log b}} + \frac{1}{\frac{\log c + \log ab}{\log c}}$

$= \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{\log c}{\log abc}$

$= \frac{\log a + \log b + \log c}{\log abc}$

$= \frac{\log abc}{\log abc}$

$= 1 = \text{RHS}$

16. if $x = 1 + \log_a bc$; $y = 1 + \log_b ca$ & $z = 1 + \log_c ab$

prove that : $xy + yz + zx = xyz$

SOLUTION

TPT : $xy + yz + zx = xyz$

TPT : $\frac{xy}{xyz} + \frac{yz}{xyz} + \frac{zx}{xyz} = 1$

TPT : $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

LHS

$= \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

$= \frac{1}{1+\log_a bc} + \frac{1}{1+\log_b ac} + \frac{1}{1+\log_c ab}$

REFER SOLN Q14

17. $x = \log_6 3$; $y = \log_9 6$; $z = \log_{12} 9$ then

prove that : $1 + xyz = 2yz$

SOLUTION

LHS

$= 1 + xyz$

$= 1 + \log_6 3 . \log_9 6 . \log_{12} 9$

$= 1 + \frac{\log 3}{\log 6} . \frac{\log 6}{\log 9} . \frac{\log 9}{\log 12}$

$= 1 + \frac{\log 3}{\log 12}$

$= \frac{\log 12 + \log 3}{\log 12}$

$= \frac{\log 36}{\log 12}$

$= \log_{12} 36$

RHS

$= 2yz$

$= 2 \log_9 6 . \log_{12} 9$

$= 2 . \frac{\log 6}{\log 9} . \frac{\log 9}{\log 12}$

$= 2 . \frac{\log 6}{\log 12}$

$= \frac{\log 6^2}{\log 12}$

$= \frac{\log 36}{\log 12}$

$= \log_{12} 36$ LHS = RHS

18.

$$\log_a x + \log_{a^2} x^2 + \log_{a^3} x^3 + \_\_\_ + \log_{a^p} x^p = p \log_a x$$

SOLUTION

LHS

$$= \log_a x + \log_{a^2} x^2 + \log_{a^3} x^3 + \_\_\_ + \log_{a^p} x^p$$

$$= \frac{\log x}{\log a} + \frac{\log x^2}{\log a^2} + \frac{\log x^3}{\log a^3} + \_\_\_ + \frac{\log x^p}{\log a^p}$$

$$= \frac{\log x}{\log a} + \frac{2 \log x}{2 \log a} + \frac{3 \log x}{3 \log a} + ---- + \frac{p \log x}{p \log a}$$

$$= \frac{\log x}{\log a} + \frac{\log x}{\log a} + \frac{\log x}{\log a} + ----- + \frac{\log x}{\log a}$$

$$= \log_a x + \log_a x + \log_a x + ---- + \log_a x$$

$$= p \log_a x \;.\; = \text{RHS}$$

19.

$$\frac{1}{\log_{10} \frac{1}{30}} + \frac{1}{\log_5 \frac{1}{30}} + \frac{1}{\log_{18} \frac{1}{30}}$$

SOLUTION

$$= \log_{1/30} 10 + \log_{1/30} 5 + \log_{1/30} 18$$

$$= \log_{1/30} 10.5.18$$

$$= \log_{1/30} 900$$

$$= \frac{\log 900}{\log (1/30)}$$

$$= \frac{\log 30^2}{\log 30^{-1}}$$

$$= \frac{2 \log 30}{-1 \log 30}$$

$$= -2$$

20. if $\log_a b + \log_c b = 2 \log_a b . \log_c b$; then prove $b^2 = ac$

SOLUTION

$$\log_a b + \log_c b = 2 \log_a b . \log_c b$$

$$\frac{\log b}{\log a} + \frac{\log b}{\log c} = 2 \frac{\log b}{\log a} . \frac{\log b}{\log c}$$

$$\cancel{\log b}\left(\frac{1}{\log a} + \frac{1}{\log c}\right) = 2 \frac{\cancel{\log b}}{\log a} . \frac{\log b}{\log c}$$

$$\frac{\textbf{log c + log a}}{\cancel{\textbf{log a . log c}}} = \frac{\textbf{2 log b}}{\cancel{\textbf{log a . log c}}}$$

$$\log ac = \log b^2$$

$$ac = b^2$$ …… PROVED

21. if $a^2 + c^2 = b^2$ then show that :

$$\frac{1}{\log_{b+c} a} + \frac{1}{\log_{b-c} a} = 2$$

SOLUTION

$$a^2 + c^2 = b^2$$

$$a^2 = b^2 - c^2$$

$$a^2 = (b + c)(b - c)$$

INSERTING LOG ON BOTH SIDES

$$\log a^2 = \log (b + c).(b - c)$$

$$2\log a = \log (b + c) + \log (b - c)$$

$$2 = \frac{\log (b + c)}{\log a} + \frac{\log (b - c)}{\log a}$$

$$2 = \log_a (b + c) + \log_a (b - c)$$

$$2 = \frac{1}{\log_{b+c} a} + \frac{1}{\log_{b-c} a}$$

………. PROVED

22. if $a^2 = b^3 = c^5 = d^6$

then show that $\log_d abc = \frac{31}{5}$

SOLUTION

$a^2 = b^3 = c^5 = d^6$

$\log a^2 = \log b^3 = \log c^5 = \log d^6$

$2 \log a = 3 \log b = 5 \log c = 6 \log d = k$

$\log a = \frac{k}{2}$, $\log b = \frac{k}{3}$; $\log c = \frac{k}{5}$ &

$\log d = \frac{k}{6}$

LHS

$= \log_d abc$

$= \frac{\log abc}{\log d}$

$= \frac{\log a + \log b + \log c}{\log d}$

$= \frac{\frac{k}{2} + \frac{k}{3} + \frac{k}{5}}{\frac{k}{6}}$

$= \frac{\cancel{k}\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5}\right)}{\frac{\cancel{k}}{6}}$

$= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{5}}{\frac{1}{6}}$

$= \frac{\frac{15 + 10 + 6}{30}}{\frac{1}{6}} = \frac{31}{5} =$ RHS

23. if $a^2 = b^3 = c^4 = d^5$

then show that $\log_a bcd = \frac{47}{30}$

SOLUTION

$a^2 = b^3 = c^4 = d^5$

$\log a^2 = \log b^3 = \log c^4 = \log d^5$

$2 \log a = 3 \log b = 4 \log c = 5 \log d = k$

$\log a = \frac{k}{2}$, $\log b = \frac{k}{3}$; $\log c = \frac{k}{4}$ &

$\log d = \frac{k}{5}$

LHS

$= \log_a bcd$

$= \frac{\log bcd}{\log a}$

$= \frac{\log b + \log c + \log d}{\log a}$

$= \frac{\frac{k}{3} + \frac{k}{4} + \frac{k}{5}}{\frac{k}{2}}$

$= \frac{\cancel{k}\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right)}{\frac{\cancel{k}}{2}}$

$= \frac{\frac{1}{3} + \frac{1}{4} + \frac{1}{5}}{\frac{1}{2}}$

$= \frac{\frac{20 + 15 + 12}{60}}{\frac{1}{2}}$

$= \frac{47}{30} =$ RHS

24. if $a^x = b^y = c^z = d^w$

then show that

$$\log_a abcd = x\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)$$

SOLUTION

$a^x = b^y = c^z = d^w$

$\log a^x = \log b^y = \log c^z = \log d^w$

$x \log a = y \log b = z \log c = w \log d = k$

$\log a = \frac{k}{x}$, $\log b = \frac{k}{y}$; $\log c = \frac{k}{z}$ &

$\log d = \frac{k}{w}$

LHS

$= \log_a abcd$

$= \dfrac{\log abcd}{\log a}$

$= \dfrac{\log a + \log b + \log c + \log d}{\log a}$

$= \dfrac{\frac{k}{x} + \frac{k}{y} + \frac{k}{z} + \frac{k}{w}}{\frac{k}{x}}$

$= \dfrac{\cancel{k}\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)}{\frac{\cancel{k}}{x}}$

$= \dfrac{\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{w}}{\frac{1}{x}}$

$= x\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)$

= RHS

25 if $a^x = b^y = c^z$ and $b^2 = ac$

then prove that $y = \dfrac{2xz}{x+z}$

SOLUTION

$a^x = b^y = c^z$

$\log a^x = \log b^y = \log c^z$

$x \log a = y \log b = z \log c = k$

$\log a = \frac{k}{x}$, $\log b = \frac{k}{y}$; $\log c = \frac{k}{z}$

NOW

$b^2 = ac$

$\log b^2 = \log ac$

$2 \log b = \log a + \log c$

$2\frac{k}{y} = \frac{k}{x} + \frac{k}{z}$

$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

$\frac{2}{y} = \frac{x + z}{xz}$

$\frac{y}{2} = \frac{xz}{x + z}$

$y = \dfrac{2xz}{x + z}$ ……. PROVED

SOLUTION TO - Q SET 5

01. $\log (x + 3) + \log (x - 3) = \log 16$

SOLUTION

$\log (x + 3).(x - 3) = \log 16$

$(x + 3)(x - 3) = 16$

$x^2 - 9 = 16$

$x^2 = 25$

$x = \pm 5$

$x = 5 \quad (\log a, a > 0)$

02. $\log (3x + 2) - \log (3x - 2) = \log 5$

SOLUTION

$\log \frac{3x + 2}{3x - 2} = \log 5$

$\frac{3x + 2}{3x - 2} = 5$

$3x + 2 = 5(3x - 2)$

$3x + 2 = 15x - 10$

$12 = 12x$

$x = 1$

03. $\log 2 + \log (x + 3) - \log (3x - 5) = \log 3$

SOLUTION

$\log \frac{2(x + 3)}{3x - 5} = \log 3$

$\frac{2(x + 3)}{3x - 5} = 3$

$2x + 6 = 9x - 15$

$6 + 15 = 9x - 2x$

$21 = 7x$

$x = 3$

04. $\log_x (8x - 3) - \log_x 4 = 2$

SOLUTION

$\log_x \frac{8x - 3}{4} = 2$

CONVERTING TO EXPONENTIAL FORM

$\frac{8x - 3}{4} = x^2$

$8x - 3 = 4x^2$

$4x^2 - 8x + 3 = 0$

$4x^2 - 6x - 2x + 3 = 0$

$2x(2x - 3) - 1(2x - 3) = 0$

$(2x - 3)(2x - 1) = 0$

$x = \frac{3}{2} \quad \text{OR} \quad x = \frac{1}{2}$

05. $\log_x 3 + \log_x 8 + \log_x 6 = 2$

SOLUTION

$\log_x (3 . 8 . 6) = 2$

$\log_x 144 = 2$

CONVERTING TO EXPONENTIAL FORM

$144 = x^2$

$x = \pm 12$

$x = 12 \quad (\log a, a > 0)$

06. $\log_8 x + \log_4 x + \log_2 x = 11$

SOLUTION

$\frac{\log x}{\log 8} + \frac{\log x}{\log 4} + \frac{\log x}{\log 2} = 11$

$\frac{\log x}{\log 2^3} + \frac{\log x}{\log 2^2} + \frac{\log x}{\log 2} = 11$

$\frac{\log x}{3\log 2} + \frac{\log x}{2 \log 2} + \frac{\log x}{\log 2} = 11$

$\frac{\log x}{\log 2}\left(\frac{1}{3} + \frac{1}{2} + 1\right) = 11$

$\frac{\log x}{\log 2}\left(\frac{2 + 3 + 6}{6}\right) = 11$

$\frac{\log x}{\log 2}\left(\frac{11}{6}\right) = 11$

$\frac{\log x}{\log 2} = 6$

$\log x = 6\log 2$

$\log x = \log 2^6$

$\log x = \log 64$

$x = 64$

07. $\log_2 x + \log_4 x + \log_{16} x = 21/4$

SOLUTION

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 4} + \frac{\log x}{\log 16} = \frac{21}{4}$$

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 2^2} + \frac{\log x}{\log 2^4} = \frac{21}{4}$$

$$\frac{\log x}{\log 2} + \frac{\log x}{2 \log 2} + \frac{\log x}{4 \log 2} = \frac{21}{4}$$

$$\frac{\log x}{\log 2}\left(1 + \frac{1}{2} + \frac{1}{4}\right) = \frac{21}{4}$$

$$\frac{\log x}{\log 2}\left(\frac{4 + 2 + 1}{4}\right) = \frac{21}{4}$$

$$\frac{\log x}{\log 2}\left(\frac{7}{4}\right) = \frac{21}{4}$$

$$\frac{\log x}{\log 2} = 3$$

$$\log x = 3 \log 2$$

$$\log x = \log 2^3$$

$$x = 8$$

08. $\log_3 x + \log_9 x + \log_{243} x = 34/5$

SOLUTION

$$\frac{\log x}{\log 3} + \frac{\log x}{\log 9} + \frac{\log x}{\log 243} = \frac{34}{5}$$

$$\frac{\log x}{\log 3} + \frac{\log x}{\log 3^2} + \frac{\log x}{\log 3^5} = \frac{34}{5}$$

$$\frac{\log x}{\log 3} + \frac{\log x}{2 \log 3} + \frac{\log x}{5 \log 3} = \frac{34}{5}$$

$$\frac{\log x}{\log 3}\left(1 + \frac{1}{2} + \frac{1}{5}\right) = \frac{34}{5}$$

$$\frac{\log x}{\log 3}\left(\frac{10 + 5 + 2}{10}\right) = \frac{34}{5}$$

$$\frac{\log x}{\log 3}\left(\frac{17}{10}\right) = \frac{34}{5}$$

$$\frac{\log x}{\log 3} = 4$$

$$\log x = 4 \log 3$$

$$\log x = \log 3^4$$

$$x = 81$$

09. $\log_{\sqrt{3}} x + \log_3 x + \log_{\sqrt{27}} x = 11$

SOLUTION

$$\log_{\sqrt{3}} x + \log_3 x + \log_{\sqrt{27}} x = 11$$

$$\frac{\log x}{\log \sqrt{3}} + \frac{\log x}{\log 3} + \frac{\log x}{\log \sqrt{27}} = 11$$

$$\frac{\log x}{\log 3^{1/2}} + \frac{\log x}{\log 3} + \frac{\log x}{\log 3^{3/2}} = 11$$

$$\frac{\log x}{\frac{1}{2}\log 3} + \frac{\log x}{\log 3} + \frac{\log x}{\frac{3}{2}\log 3} = 11$$

$$\frac{2 \log x}{\log 3} + \frac{\log x}{\log 3} + \frac{2 \log x}{3 \log 3} = 11$$

$$\frac{\log x}{\log 3}\left(2 + 1 + \frac{2}{3}\right) = 11$$

$$\frac{\log x}{\log 3}\left(\frac{6 + 3 + 2}{3}\right) = 11$$

$$\frac{\log x}{\log 3}\left(\frac{11}{3}\right) = 11$$

$$\frac{\log x}{\log 3} = 3$$

$$\log x = 3 \log 3$$

$$\log x = \log 3^3$$

$$x = 27$$

10. $2\log_{10} x = 1 + \log_{10}\left(x + \frac{11}{10}\right)$

SOLUTION

$$2\frac{\log x}{\log 10} = 1 + \frac{\log\left(x + \frac{11}{10}\right)}{\log 10}$$

$$2\frac{\log x}{\log 10} = \frac{\log 10 + \log\left(x + \frac{11}{10}\right)}{\log 10}$$

$$2 \log x = \log 10 + \log\left(x + \frac{11}{10}\right)$$

$$\log x^2 = \log 10.\left(x + \frac{11}{10}\right)$$

$$\log x^2 = \log (10x + 11)$$

$$x^2 = 10x + 11$$

$$x^2 - 10x - 11 = 0$$

$$x^2 - 11x + x - 11 = 0$$

$$x(x - 11) + 1(x - 11) = 0$$

$$(x - 11)(x + 1) = 0$$

$x = 11$ OR $x = -1$

$x = 11$ $(\log a , a > 0)$

11. $x + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$

SOLUTION

$$x + \frac{\log (1 + 2^x)}{\log 10} = \frac{x \log 5}{\log 10} + \frac{\log 6}{\log 10}$$

$$\frac{x \log 10 + \log (1 + 2^x)}{\log 10} = \frac{x \log 5 + \log 6}{\log 10}$$

$$x \log 10 + \log (1 + 2^x) = x \log 5 + \log 6$$

$$\log 10^x + \log (1 + 2^x) = \log 5^x + \log 6$$

$$\log 10^x . (1 + 2^x) = \log 5^x. 6$$

$$10^x(1 + 2^x) = 5^x.6$$

$$(5.2)^x(1 + 2^x) = 5^x.6$$

$$5^x.2^x(1 + 2^x) = 5^x.6$$

$$2^x(1 + 2^x) = 6$$

$$m(1 + m) = 6$$

$$m^2 + m - 6 = 0$$

$$m^2 + 3m - 2m - 6 = 0$$

$$m(m + 3) - 2(m + 3) = 0$$

$$(m + 3)(m - 2) = 0$$

$m = -3$ OR $m = 2$

$$m = 2$$

$$2^x = 2$$

$$x = 1$$

12. $\log_2 x + \frac{1}{2}\log_2(x+2) = 2$

SOLUTION

$\frac{2\,\text{Log}_2\, x + \log_2 (x+2)}{2} = 2$

$\log_2 x^2 + \log_2 (x+2) = 4$

$\log_2 x^2.(x+2) = 4$

CONVERTING TO EXPONENTIAL FORM

$x^2.(x+2) = 2^4.$

$x^3 + 2x^2 = 16$

$x^3 + 2x^2 - 16 = 0$

$x^3 - 8 + 2x^2 - 8 = 0$

$x^3 - 2^3 + 2(x^2 - 4) = 0$

$(x-2)(x^2 + 2x + 4) + 2(x-2)(x+2) = 0$

$(x-2)\left(x^2 + 2x + 4 + 2(x+2)\right) = 0$

$(x-2)\ (x^2 + 2x + 4 + 2x + 4) = 0$

$(x-2)\ (x^2 + 4x + 8) = 0$

$x - 2 = 0$ OR $x^2 + 4x + 8 = 0$

$B^2 - 4AC < 0$

$x = 2$ SO DISCARD

13. $\sqrt{\log_2 x^4} + 4\log_4\sqrt{\frac{2}{x}} = 2$

SOLUTION

$\sqrt{4\log_2 x} + 4\log_4\left(\frac{2}{x}\right)^{1/2} = 2$

$2\sqrt{\log_2 x} + \frac{4}{2}\log_4\left(\frac{2}{x}\right) = 2$

$2\sqrt{\log_2 x} + 2\log_4\left(\frac{2}{x}\right) = 2$

$\sqrt{\log_2 x} + \log_4 2 - \log_4 x = 1$

$\sqrt{\log_2 x} + \frac{\log 2}{\log 4} - \frac{\log x}{\log 4} = 1$

$\sqrt{\log_2 x} + \frac{\log 2}{\log 2^2} - \frac{\log x}{\log 2^2} = 1$

$\sqrt{\log_2 x} + \frac{\log 2}{2\log 2} - \frac{\log x}{2\log 2} = 1$

$\sqrt{\log_2 x} + \frac{1}{2} - \frac{1}{2}\log_2 x = 1$

$\sqrt{\log_2 x} - \frac{1}{2}\log_2 x = \frac{1}{2}$

$\sqrt{m} - \frac{1}{2}m = \frac{1}{2}$

$2\sqrt{m} - m = 1$

$2\sqrt{m} = 1 + m$

SQUARING

$4m = 1 + 2m + m^2$

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m - 1 = 0$

$m = 1$

$\log_2 x = 1$

$x = 2$

SOLUTION TO - Q SET 6

WITHOUT USING LOG TABLE

01. $\frac{1}{4} < \log_{10} 2 < \frac{1}{3}$

ASSUME

$\frac{1}{4} < \log_{10} 2$	$\log_{10} 2 < \frac{1}{3}$
$\frac{1}{4} < \frac{\log 2}{\log 10}$	$\frac{\log 2}{\log 10} < \frac{1}{3}$
$\log 10 < 4 \log 2$	$3 \log 2 < \log 10$
$\log 10 < \log 2^4$	$\log 2^3 < \log 10$
$\log 10 < \log 16$	$\log 8 < \log 10$
$10 < 16$	$8 < 10$
…. THIS IS CORRECT	…. THIS IS CORRECT

Since our assumptions are correct , we conclude

$\frac{1}{4} < \log_{10} 2 < \frac{1}{3}$

02. $\frac{2}{5} < \log_{10} 3 < \frac{1}{2}$

ASSUME

$\frac{2}{5} < \log_{10} 3$	$\log_{10} 3 < \frac{1}{2}$
$\frac{2}{5} < \frac{\log 3}{\log 10}$	$\frac{\log 3}{\log 10} < \frac{1}{2}$
$2\log 10 < 5 \log 3$	$2 \log 3 < \log 10$
$\log 10^2 < \log 3^5$	$\log 3^2 < \log 10$
$\log 100 < \log 243$	$\log 9 < \log 10$
$100 < 243$	$9 < 10$
…. THIS IS CORRECT	…. THIS IS CORRECT

Since our assumptions are correct , we conclude

$\frac{2}{5} < \log_{10} 3 < \frac{1}{2}$

03. $\frac{3}{10} < \log_{10} 2 < \frac{1}{3}$

ASSUME

$\frac{3}{10} < \log_{10} 2$	$\log_{10} 2 < \frac{1}{3}$
$\frac{3}{10} < \frac{\log 2}{\log 10}$	$\frac{\log 2}{\log 10} < \frac{1}{3}$
$3\log 10 < 10 \log 2$	$3 \log 2 < \log 10$
$\log 10^3 < \log 2^{10}$	$\log 2^3 < \log 10$
$\log 1000 < \log 1024$	$\log 8 < \log 10$
$1000 < 1024$	$8 < 10$
…. THIS IS CORRECT	…. THIS IS CORRECT

Since our assumptions are correct , we conclude

$\frac{3}{10} < \log_{10} 2 < \frac{1}{3}$

04. $\frac{2}{3} < \log_{10} 5 < \frac{3}{4}$

ASSUME

$\frac{2}{3} < \log_{10} 5$	$\log_{10} 5 < \frac{3}{4}$
$\frac{2}{3} < \frac{\log 5}{\log 10}$	$\frac{\log 5}{\log 10} < \frac{3}{4}$
$2\log 10 < 3 \log 5$	$4 \log 5 < 3\log 10$
$\log 10^2 < \log 5^3$	$\log 5^4 < \log 10^3$
$\log 100 < \log 125$	$\log 625 < \log 1000$
$100 < 125$	$625 < 1000$
…. THIS IS CORRECT	…. THIS IS CORRECT

Since our assumptions are correct , we conclude

$\frac{2}{3} < \log_{10} 5 < \frac{3}{4}$

SOLUTION TO - EXTRA Q'S

01. $\dfrac{\log\sqrt{8} + \log\sqrt{27} - \log\sqrt{125}}{\log 6 - \log 5} = \dfrac{3}{2}$

$$= \frac{\log\sqrt{2^3} + \log\sqrt{3^3} - \log\sqrt{5^3}}{\log 6 - \log 5}$$

$$= \frac{\log 2^{3/2} + \log 3^{3/2} - \log 5^{3/2}}{\log 6 - \log 5}$$

$$= \frac{\frac{3}{2}\log 2 + \frac{3}{2}\log 3 - \frac{3}{2}\log 5}{\log 6 - \log 5}$$

$$= \frac{\frac{3}{2}\left(\log 2 + \log 3 - \log 5\right)}{\log 6 - \log 5}$$

$$= \frac{3}{2}\ \frac{\log 6 - \log 5}{\log 6 - \log 5}$$

$$= \frac{3}{2} = \text{RHS}$$

02. EVALUATE

$$\log_2\left(1 + \frac{1}{2}\right) + \log_2\left(1 + \frac{1}{3}\right) + \log_2\left(1 + \frac{1}{4}\right) + \ldots\ldots + \log_2\left(1 + \frac{1}{127}\right)$$

$$= \log_2\left(\frac{3}{2}\right) + \log_2\left(\frac{4}{3}\right) + \log_2\left(\frac{5}{4}\right) + \ldots + \log_2\left(\frac{128}{127}\right)$$

$$= \log_2\left(\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots\ldots \times \frac{128}{127}\right)$$

$$= \log_2\left(\frac{128}{2}\right)$$

$$= \log_2 64$$

$$= \log_2 2^6$$

$$= 6\log_2 2$$

$$= 6$$

03. if $x = \dfrac{e^y - e^{-y}}{e^y + e^{-y}}$

SHOW THAT : $y = \dfrac{1}{2}\log_e\left(\dfrac{1 + x}{1 - x}\right)$

SOLUTION

$$\frac{1 + x}{1 - x} = \frac{1 + \frac{e^y - e^{-y}}{e^y + e^{-y}}}{1 + \frac{e^y - e^{-y}}{e^y + e^{-y}}}$$

$$= \frac{\frac{e^y + e^{-y} + e^y - e^{-y}}{e^y + e^{-y}}}{\frac{e^y + e^{-y} - e^y + e^{-y}}{e^y + e^{-y}}}$$

$$= \frac{2e^y}{2e^{-y}}$$

$$= e^{y + y}$$

$$= e^{2y}$$

Now ; $\dfrac{1 + x}{1 - x} = e^{2y}$

TAKING LOG (TO THE BASE 'e') ON BOTH SIDES

$$\log_e\left(\frac{1 + x}{1 - x}\right) = \log_e e^{2y}$$

$$\log_e\left(\frac{1 + x}{1 - x}\right) = 2y \log_e e$$

$$\log_e\left(\frac{1 + x}{1 - x}\right) = 2y$$

$$y = \frac{1}{2}\log_e\left(\frac{1 + x}{1 - x}\right)$$

04.
$$\frac{3 + \log_{10} 343}{2 + \frac{1}{2}\log_{10}\left(\frac{49}{4}\right) + \frac{1}{2}\log_{10}\left(\frac{1}{25}\right)}$$

$$= \frac{3 + \log_{10} 7^3}{2 + \log_{10}\sqrt{\frac{49}{4}} + \log_{10}\sqrt{\frac{1}{25}}}$$

$$= \frac{3 + 3\log_{10} 7}{2 + \log_{10}\frac{7}{2} + \log_{10}\frac{1}{5}}$$

$$= \frac{3 + 3\log_{10} 7}{2 + \log_{10}\left(\frac{7}{2}\ \frac{1}{5}\right)}$$

$$= \frac{3 + 3\log_{10} 7}{2 + \log_{10}\left(\frac{7}{10}\right)}$$

$$= \frac{3 + 3\log_{10} 7}{2 + \log_{10} 7 - \log_{10} 10}$$

$$= \frac{3 + 3\log_{10} 7}{2 + \log_{10} 7 - 1}$$

$$= \frac{3\left(1 + \log_{10} 7\right)}{1 + \log_{10} 7}$$

$$= 3$$

PAPER - II

LOGARITHMS

SUMS ON LOG TABLES

01. $\frac{25 \times 43}{350}$

$a = \frac{25 \times 43}{350}$

log a = log 25 + log 43 – log 350

log a = 1.3979 + 1.6335 – 2. 5441

log a = 3.0314 – 2.5441

log a = 0.4873

a = AL(0.4873)

a = 3.071

02. $\frac{573 \times 624}{7293}$

$a = \frac{573 \times 624}{7293}$

log a = log 573 + log 624 – log 7293

log a = 2.7582 + 2.7952 – 3.8629

log a = 5.5534 – 3.8629

log a = 1.6905

a = AL(1.6905)

a = 49.04 BY CALC : 49.03

03. $a = \frac{(2.41)^2 \times 2.61}{1.374}$

log a = 2.log 2.41 + log 2.61 – log 1.374

log a = 2(0.3820) + 0.4166 – 0.1380

log a = 0.7640 + 0.4166 – 0.1380

log a = 1.1806 – 0.1380

log a = 1.0426

a = AL(1.0426)

a = 11.04 BY CAL : 11.0328

04. $a = \frac{(3.54)^3 \times (1.34)^2}{75.54}$

log a = 3 log 3.54 + 2 log 1.34 – log 75.54

log a = 3(0.5490) + 2(0.1271) – 1.8781

log a = 1.6470 + 0.2542 – 1.8781

log a = 1.9012 – 1.8781

log a = 0.0231

a = AL(0.0231)

a = 1.054 BY CAL : 1.05448

05. Given $\pi = 3.142$, $r = 2.307$, $h = 8.5$. Find the value of V where $V = \pi r^2 h$

$V = \pi r^2 h$

$V = 3.142 \times (2.307)^2 \times 8.5$

$\text{Log } V = \log 3.142 + 2\log 2.307 + \log 8.5$

$\text{Log } V = 0.4972 + 2(0.3630) + 0.9294$

$\text{Log } V = 0.4972 + 0.7260 + 0.9294$

$\text{Log } V = 2.1526$

$V = AL(2.1526)$

$V = 142.1$

06. if the area of circle is 88.2 sq.m and $\pi = 3.142$, find r

$A = \pi r^2$

$88.2 = 3.142\, r^2$

$r^2 = \dfrac{88.2}{3.142}$

$r = \sqrt{\dfrac{88.2}{3.142}}$

$\log r = \dfrac{1}{2}\left[\log 88.2 - \log 3.142\right]$

$\log r = \dfrac{1}{2}\left[1.9455 - 0.4972\right]$

$\log r = \dfrac{1.4483}{2}$

$r = AL\,(0.7242) = 5.299$

07. Find radius of the sphere whose volume is 500 cm3 by using log table

$V = \dfrac{4}{3} \pi r^3$

$500 = \dfrac{4}{3} \times 3.142 \times r^3$

$r^3 = \dfrac{500 \times 3}{4 \times 3.142}$

$r = \left(\dfrac{1500}{12.568}\right)^{1/3}$

$\log r = \dfrac{1}{3}\left[\log 1500 - \log 12.568\right]$

$\log r = \dfrac{1}{3}\left[3.\ 1761 - 1.0993\right]$

$\log r = \dfrac{2.0768}{3}$

$\log r = 0.6923$

$r = AL(0.6923)$

$r = 4.923$

08. the population of a town at present is 80000 . If the annual rate of increase is 4% , find the population after 4 years

Use the formula : $A = P\left(1 + \frac{r}{100}\right)^n$

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$A = 80000 \left(1 + \frac{4}{100}\right)^4$$

A = 80000 x (1.04)4

Log A = log 80000 + 4log 1.04

Log A = 4.9031 + 4(0.0170)

Log A = 4.9031 + 0.0680

Log A = 4.9711

A = AL(4.9711)

A = 93560

9) SIMPLIFY

a) $\bar{3}.5472 - \bar{2}.8371 + 1.4581$

$$\begin{array}{rr} & \bar{4} \\ & \not{\bar{3}}.5472 \\ - & \bar{2}.8371 \\ \hline -4 + 2 \longrightarrow & \bar{2}.7101 \end{array}$$

$= \bar{2}.7101 + 1.4581$

CARRY 1

$$\begin{array}{rr} & \bar{2}.7101 \\ + & 1.4581 \\ \hline -2 + 2 \longrightarrow & 0.1682 \end{array}$$

b) $1.2489 - \bar{1}.0891 + \bar{2}.8897$

$$\begin{array}{rr} & 1.2489 \\ - & \bar{1}.0891 \\ \hline 1 + 1 \longrightarrow & 2.1598 \end{array}$$

$= 2.1598 + \bar{2}.8897$

CARRY 1

$$\begin{array}{rr} & \bar{2}.1598 \\ + & 2.8897 \\ \hline -2 + 3 \longrightarrow & 1.0495 \end{array}$$

c) $1.5548 + \bar{2}.7110 - \bar{2}.7619$

CARRY 1

$$\begin{array}{rr} & 1.5548 \\ + & \bar{2}.7110 \\ \hline 2 - 2 \longrightarrow & 0.2658 \end{array}$$

$= 0.2658 - \bar{2}.7619$

$$\begin{array}{rr} & \bar{1}\ \not{0}.2658 \\ - & \bar{2}.7619 \\ \hline -1 + 2 \longrightarrow & 1.5039 \end{array}$$

d) $\bar{6}.9666 - \bar{1}.7965 - 0.1832$

CARRY 1

$$\begin{array}{rr} & \bar{6}.9666 \\ - & \bar{1}.7965 \\ \hline -6 + ! \longrightarrow & \bar{5}.1701 \end{array}$$

$= \bar{5}.1701 - 0.1832$

$$\begin{array}{rr} & \bar{6}\ \not{\bar{5}}.1701 \\ - & 0.1832 \\ \hline -6 - 0 \longrightarrow & \bar{6}.9869 \end{array}$$

PAPER – II : LOGARITHMS

10. $a = \dfrac{45.83 \times 0.5432}{0.02739}$

$\log a = \log 45.83 + \log 0.5432 - \log 0.02739$

$\log a = 1.6612 + \bar{1}.7350 - \bar{2}.4376$

$\log a = 1.3962 - \bar{2}.4376$

$\log a = 2.9586$

$a = AL(2.9586)$

$a = 909.0$ BY CALC : 908.9

11. $a = \sqrt{\dfrac{35.87 \times 0.0514}{0.0578}}$

$\log a = \dfrac{1}{2}\left(\log 35.87 + \log 0.0514 - \log 0.0578\right)$

$\log a = \dfrac{1}{2}\left(1.5548 + \bar{2}.7110 - \bar{2}.7619\right)$

$\log a = \dfrac{1}{2}\left(0.2658 - \bar{2}.7619\right)$

$\log a = \dfrac{1}{2}\left(1.5039\right)$

$\log a = 0.75195$

$\log a = 0.7520$

$a = AL(0.7520) = 5.649$ BY CALC : 5.648

12. $x = \sqrt[4]{\dfrac{(72.14)^5 \times \sqrt{45}}{(2.8)^3 \times \sqrt{32}}}$

$\log x = \frac{1}{4}\left(5 \log 72.14 + \frac{1}{2}\log 45 - 3\log 2.8 - \frac{1}{2}\log 32\right)$

$\log x = \frac{1}{4}\left(5(1.8581) + \frac{1}{2}(1.6532) - 3(0.4472) - \frac{1}{2}(1.5051)\right)$

$\log x = \frac{1}{4}\left(9.2905 + 0.8266 - 1.3416 - 0.7526\right)$

$\log x = \frac{1}{4}\left(10.1171 - 1.3416 - 0.7526\right)$

$\log x = \frac{1}{4}\left(8.7755 - 0.7526\right)$

$\log x = \frac{1}{4}\left(8.0229\right)$

$\log x = 2.005725$

$\log x = 2.0057$

$x = AL(2.0057)$

$= 101.4$

BY CALC : 101.358

13. $a = \sqrt{\dfrac{0.021^3}{0.6258 \times \sqrt[5]{8.24}}}$

$\log a = \frac{1}{2}\left(3\log 0.021 - \log 0.6258 - \frac{1}{5}\log 8.24\right)$

$\log a = \frac{1}{2}\left(3(\bar{2}.3222) - \bar{1}.7965 - \frac{0.9159}{5}\right)$

$\log a = \frac{1}{2}\left(\bar{6}.9666 - \bar{1}.7965 - 0.1832\right)$

$\log a = \frac{1}{2}\left(\bar{5}.1701 - 0.1832\right)$

$\log a = \dfrac{\bar{6}.9869}{2}$

$\log a = \bar{3}.4935$

$a = AL(\bar{3}.4935)$

$a = 0.003116$

BY CALC : 0.0031154

14. $a = \dfrac{(0.2346)^2 \times \sqrt[3]{772.7}}{(12.45)^3 \times \sqrt{0.000382}}$

$\log a = 2 \log 0.2346 + \frac{1}{3} \log (772.7) - 3 \log 12.45 - \frac{1}{2} \log 0.000382$

$\log a = 2(\bar{1}.3703) + \frac{1}{3}(2.8880) - 3(1.0951) - \frac{\bar{4}.5821}{2}$

$\log a = \bar{2}.7406 + 0.9627 - 3.2853 - \bar{2}.2911$

$\log a = \bar{1}.7033 - 3.2853 - \bar{2}.2911$

$\log a = \bar{4}.4180 - \bar{2}.2911$

$\log a = \bar{2}.1269$

$a = AL(\bar{2}.1269)$

$a = 0.01340$ BY CALC : 0.01339

15. $a = \sqrt[3]{\dfrac{16.23}{426.8}}$

$\log a = \frac{1}{3}\left(\log 16.23 - \log 426.8\right)$

$\log a = \frac{1}{3}\left(1.2103 - 2.6302\right)$

$\log a = \frac{1}{3}\left(\bar{2}.5801\right)$

$= \dfrac{\bar{2}.5801}{3}$

$= \dfrac{\bar{2} + 0.5801}{3}$

$= \dfrac{\bar{3} + 1.5801}{3}$

$= \dfrac{\bar{3}}{3} + \dfrac{1.5801}{3}$

$= \bar{1} + 0.5267$

$\log a = \bar{1}.5267$

$a = AL(\bar{1}.5267)$

$a = 0.3362$ BY CALC : 0.33627

16.

$$a = \frac{(0.3125)^2}{(0.4629)^{1/3}}$$

$$\log a = 2\log(0.3125) - \frac{1}{3}\log(0.4629)$$

$$\log a = 2(\bar{1}.4949) - \frac{1}{3}(\bar{1}.6654)$$

CALC. $\frac{\bar{1}.6654}{3}$

$$= \frac{\bar{1} + 0.6654}{3}$$

$$= \frac{\bar{3} + 2.6654}{3}$$

$$= \frac{\bar{3}}{3} + \frac{2.6654}{3}$$

$$= \bar{1} + 0.88846$$

$$= \bar{1}.8885$$

$$\log a = \bar{1}.1013$$

$$a = AL(\bar{1}.1013)$$

$$a = 0.1263$$

BY CALC : 0.12624

17. $$a = \frac{93.652 \times \sqrt[4]{0.008}}{(2.382)^3}$$

$$\log a = \log 93.652 + \frac{1}{4}\log 0.008 - 3\log 2.382$$

$$\log a = 1.9715 + \frac{1}{4}\,\bar{3}.9031 - 3(0.3770)$$

CALC $\frac{\bar{3}.9031}{4}$

$$= \frac{\bar{3} + 0.9031}{4}$$

$$= \frac{\bar{4} + 1.9031}{4}$$

$$= \frac{\bar{4}}{4} + \frac{1.9031}{4}$$

$$= \bar{1} + 0.4758$$

$$= \bar{1}.4758$$

$$\log a = 1.9715 + \bar{1}.4758 - 1.1310$$

$$\log a = 1.4473 - 1.1310$$

$$\log a = 0.3163$$

$$a = AL(0.3163)$$

$$a = 2.071$$

BY CALC : 2.0723

18. $a = \dfrac{0.0543 \times (974.2)^2}{\sqrt[3]{0.1234}}$

$\log a = \log 0.0543 + 2 \log 974.2 - \dfrac{1}{3} \log 0.1234$

$\log a = \bar{2}.7348 + 2(2.9887) - \dfrac{1}{3}(\bar{1}.0913)$

CALC $\dfrac{\bar{1}.0913}{3}$

$= \dfrac{\bar{1} + 0.0913}{3}$

$= \dfrac{\bar{3} + 2.0913}{3}$

$= \dfrac{\bar{3}}{3} + \dfrac{2.0913}{3}$

$= \bar{1} + 0.6971$

$= \bar{1}.6971$

$\log a = \bar{2}.7348 + 5.9774 - \bar{1}.6971$

$\log a = 4.7122 - \bar{1}.6971$

$\log a = 5.0151$

$a = AL(5.0151)$

$a = 103500$

BY CALC : 103512.08

19. $a = \dfrac{28.45 \times \sqrt[3]{0.3254}}{32.43 \times \sqrt[5]{0.3046}}$

$\log a = \log 28.45 + \dfrac{1}{3} \log 0.3254 - \log 32.43 - \dfrac{1}{5} \log 0.3046$

$\log a = 1.4541 + \dfrac{1}{3}(\bar{1}.5124) - 1.5109 - \dfrac{1}{5}(\bar{1}.4838)$

CALC $\dfrac{\bar{1}.5124}{3}$ $\qquad$ $\dfrac{\bar{1}.4838}{5}$

$= \dfrac{\bar{1} + 0.5124}{3}$ $\qquad$ $= \dfrac{\bar{1} + 0.4838}{5}$

$= \dfrac{\bar{3} + 2.5124}{3}$ $\qquad$ $= \dfrac{\bar{5} + 4.4838}{5}$

$= \dfrac{\bar{3}}{3} + \dfrac{2.5124}{3}$ $\qquad$ $= \dfrac{\bar{5}}{5} + \dfrac{4.4838}{5}$

$= \bar{1} + 0.83746$ $\qquad$ $= \bar{1} + 0.89676$

$= \bar{1}.8375$ $\qquad$ $= \bar{1}.8968$

$\log a = 1.4541 + \bar{1}.8375 - 1.5109 - \bar{1}.8968$

$\log a = 1.2916 - 1.5109 - \bar{1}.8968$

$\log a = \bar{1}.7807 - \bar{1}.8968$

$\log a = \bar{1}.8839$

$a = AL(\bar{1}.8839)$

$a = 0.7654$

BY CALC : 0.76535

20. $a = \dfrac{27.38 \times \sqrt[3]{0.3052}}{31.65 \times \sqrt[5]{0.3028}}$

$\log a = \log 27.38 + \frac{1}{3} \log 0.3052 - \log 31.65 - \frac{1}{5} \log 0.3028$

$\log a = 1.4375 + \frac{1}{3}(\bar{1}.4846) - 1.5004 - \frac{1}{5}(\bar{1}.4811)$

CALC $\dfrac{\bar{1}.4846}{3}$ $\qquad$ $\dfrac{\bar{1}.4811}{5}$

$= \dfrac{\bar{1} + 0.4846}{3}$ $\qquad$ $= \dfrac{\bar{1} + 0.4811}{5}$

$= \dfrac{\bar{3} + 2.4846}{3}$ $\qquad$ $= \dfrac{\bar{5} + 4.4811}{5}$

$= \dfrac{\bar{3}}{3} + \dfrac{2.4846}{3}$ $\qquad$ $= \dfrac{\bar{5}}{5} + \dfrac{4.4811}{5}$

$= \bar{1} + 0.8282$ $\qquad$ $= \bar{1} + 0.89622$

$= \bar{1}.8282$ $\qquad$ $= \bar{1}.8962$

$\log a = 1.4375 + \bar{1}.8282 - 1.5004 - \bar{1}.8962$

$\log a = 1.2657 - 1.5004 - \bar{1}.8962$

$\log a = \bar{1}.7653 - \bar{1}.8962$

$\log a = \bar{1}.8691$

$a = AL(\bar{1}.8691)$

$a = 0.7398$ $\qquad$ BY CALC : 0.7396

21. if $\log_{10}3 = 0.4771212$,
without using log tables , find

a) $\log_{10}9 = \log_{10}3^2$

$= 2\log_{10}3$

$= 2(0.4771212) = 0.9542424$

b) $\log_{10}\sqrt{3} = \frac{1}{2} \log_{10}3$

$= \dfrac{0.4771212}{2} = 0.2385606$

c) $\log_{10}\left(\frac{1}{9}\right) = \log_{10}9^{-1}$

$= \log_{10}3^{-2}$

$= -2 \cdot \log_{10}3$

$= -2(0.4771212) = -0.9542424$

d) $\log_{10}(0.3) = \log_{10}\left(\frac{3}{10}\right)$

$= \log_{10}3 - \log_{10}10$

$= 0.4771212 - 1$

$= \bar{1}.4771212$

22. If log 33.48 = 1.5247854 ; find

a) $\log \sqrt[3]{33.48} = \frac{1}{3} \log 33.48$

$= \frac{1.5247854}{3} = 0.5082618$

b) log 334800 = 5.5247854

c) antilog 4.5247854 = 33480

23. if $\log_{10} 2 = 0.3010$, find the number of digits in 2^{64}

$a = 2^{64}$

log a = 64 log2

log a = 64(0.3010)

log a = 19.264

a = AL(19.264)

Hence number of digits in 2^{64} = 19 + 1 = 20